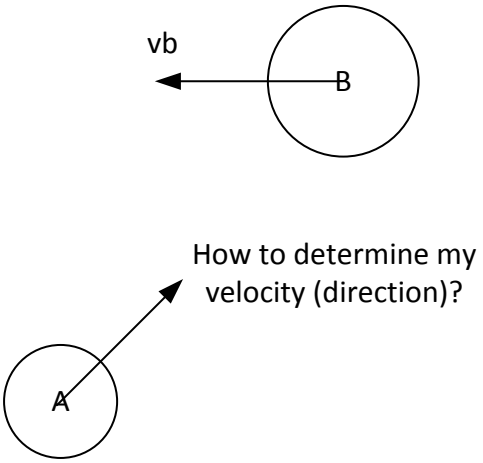
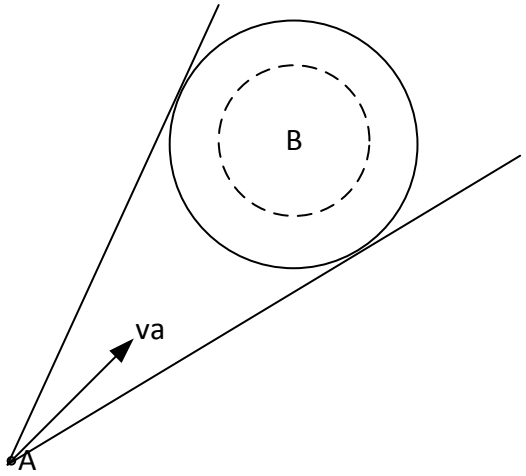
Problem statement: suppose agent B is moving at a constant velocity v_b (both the direction and magnitude are constant). Agent A is also moving at a constant speed, then determine at what velocities agent A will collide with agent B.



First, it is important to consider the special case of $v_b = 0$.

In this case, if the velocity v_A is inside the cone determined by the sphere centered at B with radius as $r_A + r_B$, then there exists a time t such that agent A will collide with B. In other words, if v_A is not in this cone, then they will never collide. Note that no matter what the velocity magnitude is, as long as the velocity direction is in this cone, it will collide. The magnitude determines how long it will take before collision.

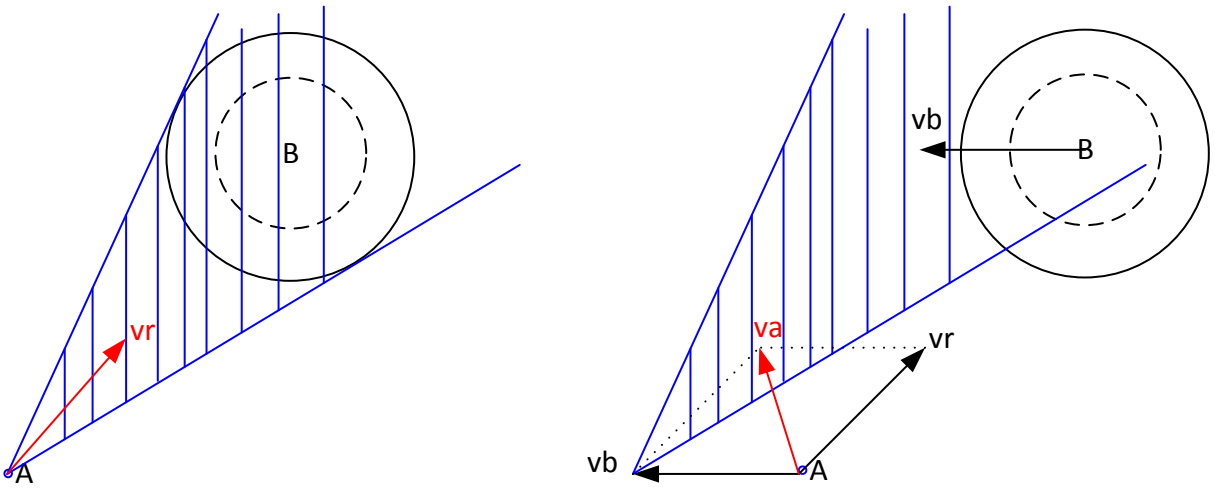


Now we consider the general case of $v_B \neq 0$.

In this case, we convert it back to the above simple case by considering the relative motion. The relative velocity of A with respect to B is

$$v_r = v_A - v_B$$

Again, if the relative velocity is in the cone, the two agents will collide! So the cone determines the forbidden relative velocity. Since $v_A = v_r + v_B$, the forbidden v_A is the cone translated by v_B as shown below.



The mathematical definition of velocity obstacle is

$$VO_{A|B} = \{\mathbf{v} \mid \exists t > 0 : (\mathbf{v} - \mathbf{v}_B)t \in D(\mathbf{x}_B - \mathbf{x}_A, r_A + r_B)\}$$

where A has position $\mathbf{x}_A$ and radius r_A, and B has position $\mathbf{x}_B$, radius r_B, and velocity $\mathbf{v}_B$. The notation $D(\mathbf{x}, r)$ represents a disc with center $\mathbf{x}$ and radius r.

How to interpret different velocities in the velocity obstacle? The short answer is that different velocity will lead to different collision time and collision position.

For example, for the three velocities as shown below, we can consider the corresponding relative velocity. Then, the answer is clear

- For the case of v_{A1}, it will take infinite time to collide, because the relative velocity magnitude is zero
- For the case of v_{A2}, when agent A and B collide, they will just have one overlapping point, because the corresponding relative velocity is tangent to the obstacle. Similar to velocity v_{A3}.